Theory of the sub-Poisonnian distribution of the Rydberg excitation (squeezed state)

Theoretical approch introcing a Dicke description of the laser excitation to treat the dynamics and the statistics of the Rydberg excitation

Collective Dicke states

- We introduce the collective Dicke to treat the laser excitation excitation of *N* atoms.
- There is no difference with the approach of a two-level system, but with have a direct access to the statistics of the excitation
- Each atom is described as a spin ½, we consider the sum of the ensemble of these spins to introduce giant dipoles
- Dicke introduces a cooperative number *r* (varying from *N*/2 and 0 or ½) and a difference of population number *m* (varying from –*N*/2 and +*N*/2)
- For two atoms, we have the symmetrical states and the antisymmetrical state

$$\left|r=1,m=+1\right\rangle=\left|r,r\right\rangle \;;\; \left|r=1,m=0\right\rangle=\left(\left|g,r\right\rangle+\left|r,g\right\rangle\right)\Big/\sqrt{2} \;;\; \left|r=1,m=-1\right\rangle=\left|g,g\right\rangle$$

$$\left|r=0,m=0\right\rangle=\left(\left|g,r\right\rangle-\left|r,g\right\rangle\right)\Big/\sqrt{2}$$

Dicke collective operators

Dicke states *r=N/2,N/2-1…1/2 or 0 ; -r ≤ m ≤r*

$$R^{+}_{\vec{k}} = \sum_{i=1}^{N} |2,i\rangle\langle 1,i| \exp\left(i\vec{k}.\vec{\hat{R}}_i\right)$$

$$R^{-}_{\vec{k}} = \sum_{i=1}^{N} |1,i\rangle\langle 2,i| \exp\left(-i\vec{k}.\vec{\hat{R}}_i\right)$$

$$R^{(3)}_{\vec{k}} = \frac{1}{2}\sum_{i=1}^{N} \left[|2,i\rangle\langle 2,i| - |1,i\rangle\langle 1,i|\right]$$

$$\left[R^{+}_{\vec{k}_0}, R^{-}_{\vec{k}_0}\right] = 2R^{(3)}_{\vec{k}_0}$$

$$\left[R^{+}_{\vec{k}_0}, R^{(3)}_{\vec{k}_0}\right] = 2R^{+}_{\vec{k}_0}$$

$$R^{2}_{\vec{k}_0} = R^{(3)2}_{\vec{k}_0} + \frac{1}{2}\left[R^{+}_{\vec{k}_0} R^{-}_{\vec{k}_0} + R^{-}_{\vec{k}_0} R^{+}_{\vec{k}_0}\right]$$

$$R^{(3)}_{\vec{k}_0} |r,m,\alpha\rangle = m\,|r,m,\alpha\rangle$$

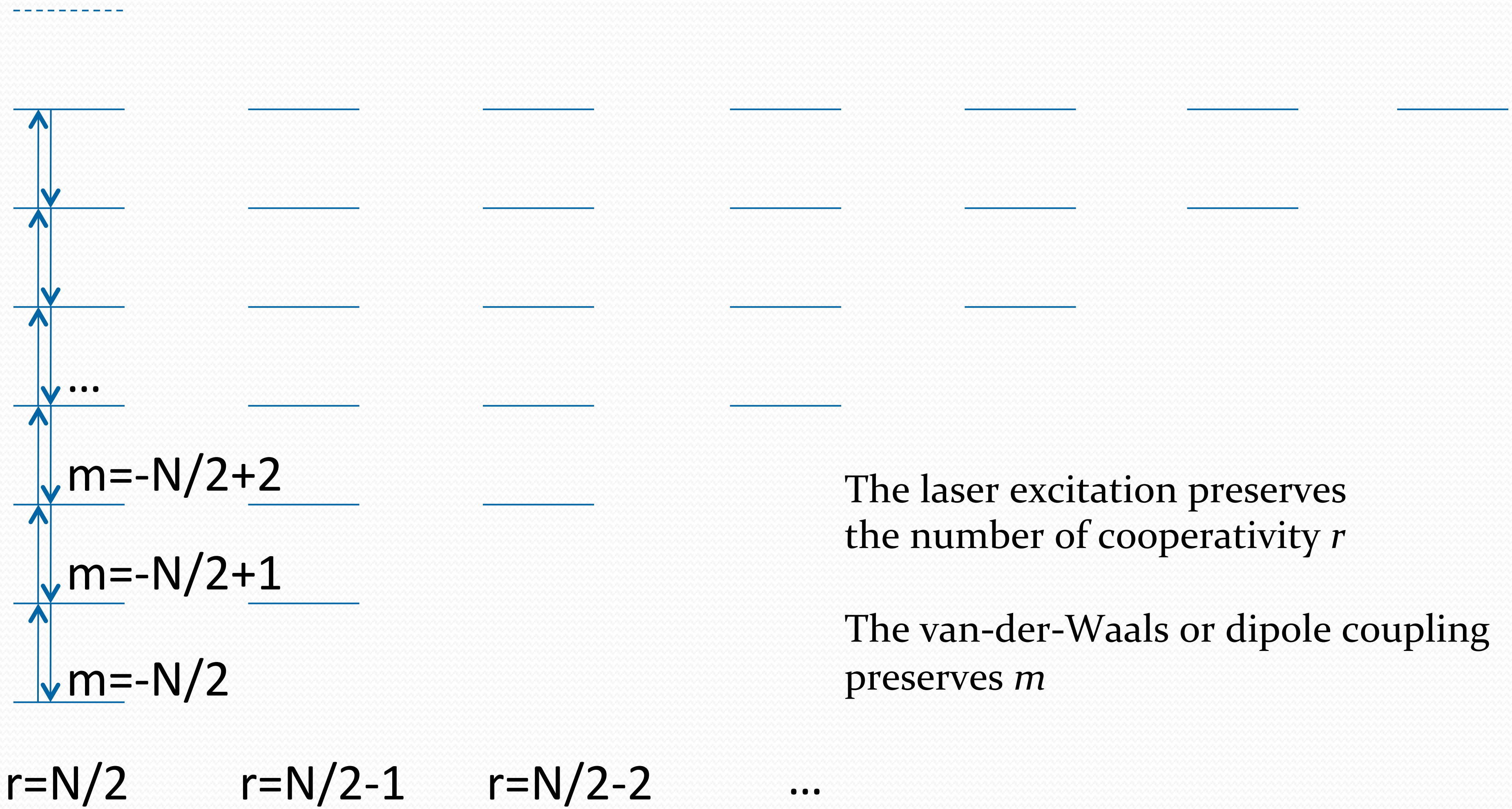
$$R^{2}_{\vec{k}_0} |r,m,\alpha\rangle = r(r+1)\,|r,m,\alpha\rangle$$

Degeneracy

$$C_N^{N/2-r} - C_N^{N/2-r+1}$$

$$\left\langle r=\frac{N}{2}, m \right| R^{\pm}_{\vec{k}_0} \left| r=\frac{N}{2}, m' \right\rangle = \sqrt{(r\pm m)(r\mp m+1)}\,\delta_{m',m\pm 1}$$

Dicke states



The laser excitation preserves the number of cooperativity *r*

The van-der-Waals or dipole coupling preserves *m*

Use of fully symmetrical Dicke states only

- We introduce the number of excited Rydberg atoms *j*

$$\Psi(t) = \sum_{m=-N/2}^{N/2} a_m(t)\left|r = N/2, m, \alpha = 1\right\rangle = \sum_{j=0}^{N} a_j(t)\left|j, sym\right\rangle$$

$$j = -N/2 + m$$

$$i\frac{d}{dt}a_j(t) = -j\delta a_j(t) + \sqrt{(N-j)(j+1)}\,\frac{\Omega}{2}a_{j+1}(t) + \sqrt{(N-j+1)j}\,\frac{\Omega}{2}a_{j-1}(t)$$

- We see the introduction of the collective Rabi fequencies in the *N* equations

Van der Waals blockade mean-field equations

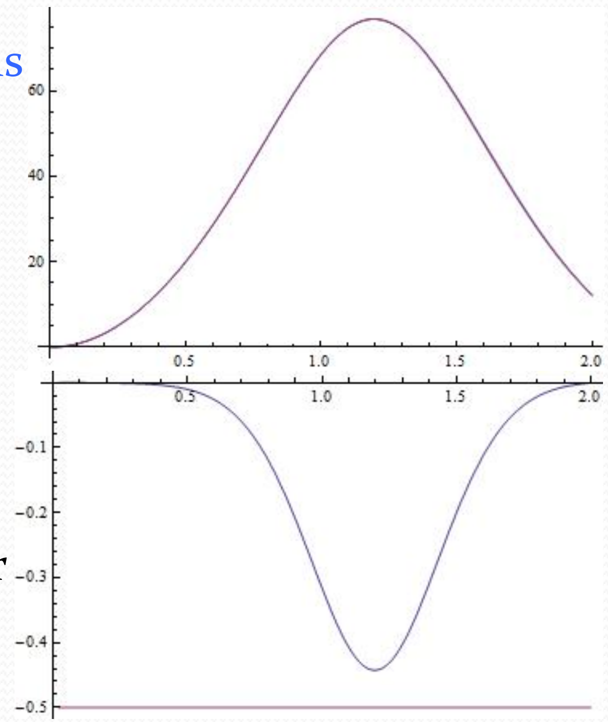
$$i\frac{d}{dt}a_j\left(t\right) = -j\delta a_j\left(t\right) + \frac{j(j+1)}{2}\frac{\eta}{N-1}a_j\left(t\right) + \sqrt{(N-j)(j+1)}\,\frac{\Omega}{2}a_{j+1}\left(t\right) + \sqrt{(N-j+1)\,j}\,\frac{\Omega}{2}a_{j-1}\left(t\right)$$

with $$\left\langle r=\frac{N}{2}, m=\frac{N}{2}-j\left|\sum_{k\neq l} W_{kl}\right| r=\frac{N}{2}, m=\frac{N}{2}-j\right\rangle = \frac{j\left(j-1\right)/2}{N-1}\eta$$

To estimate the interatomic van der Waals
Coupling we average on all the pairs of atoms

$71d_{5/2}$
Density ≈ 1.2 10^{10} at.cm^{-3}
Atoms ≈ 8 10^{3}
Rabi 1 atom ≈ 45 kHz

No difference with the two-level
approach, but we have access
to the statistics distribution



To go further: we need to consider the non-fully symmetrical states

- Why? The rate of blockade presents a non-observed asymmetrical profiles versus the laser detuning…
- But the Dicke states are not well adapted to treat the van der Waals coupling
- We need to consider a basis which diagonalizes the van der Waals coupling to the restriction of the space of the non fully symmetrical Dicke states ($r \neq N/2$)
- Such a basis exists, but is no longer adapted for describing the laser excitation
- Therefore we neglect the laser excitation of the non-symmetrical states at least in a first step

A basis for non symmetrical states to diagonalize the van-der-Waals hamiltonian W ~ C_6/R^6

$$\Psi_j(t) = \sum_{r,\alpha} b_{r,\alpha}^{(j)}(t) \left| r, m = j - N/2, \alpha \right\rangle = \sum_{j=0}^{N} c_q^{(j)}(t) \left| j, q, W \right\rangle$$

We use the van-der-Waals basis $\{j,q,W\}$ for building an ortho-normed basis by projecting them on the Dicke state

$$\left| j, sym \right\rangle = \left| r = N/2, m = j - r \right\rangle$$

$$\left| j, \tilde{q}, W \right\rangle = \left| j, q, W \right\rangle + \frac{1}{\sqrt{M} - 1} \left| j, q_M, W \right\rangle \quad \text{with} \quad \left\langle j, q_M, W \middle| W \middle| j, q_M, W \right\rangle = \left\langle j, sym \middle| W \middle| j, sym \right\rangle$$

$$\left| j, q', W \right\rangle = \left| j, \tilde{q}, W \right\rangle - \left\langle j, sym \middle| W \middle| j, \tilde{q}, W \right\rangle \left| j, sym \right\rangle \qquad M = C_N^j$$

$$\left\langle j, q', W \middle| j, sym \right\rangle = 0 \qquad \left\langle j, sym \middle| j, sym \right\rangle = 1$$

1 symmetrical state

$$\left\langle j, q', W \middle| j, p', W \right\rangle = 0 \qquad \left\langle j, q', W \middle| j, q', W \right\rangle = 1$$

M-1 van der Waals states

Van-der-Waals or dipole coupling *W*

$$\left\langle j,q',W \middle| W \middle| j,q',W \right\rangle = W_{q'q'} = W_{qq}$$

$$\left\langle j,p',W \middle| W \middle| j,q',W \right\rangle = 0$$

$$\left\langle j,sym \middle| W \middle| j,q',W \right\rangle = W_{sq'} = \left[W_{qq} - W_{ss} \right] \frac{1}{\sqrt{M}}$$

New set of equations

$$i\frac{d}{dt}b_{j,q'}\left(t\right) = -j\delta b_{j,q'}\left(t\right) + W_{q's}a_j\left(t\right)$$

$$i\frac{d}{dt}a_j\left(t\right) = -j\delta a_j\left(t\right) + W_{ss}a_j\left(t\right) + \sqrt{(N-j)(j+1)}\frac{\Omega}{2}a_{j+1}\left(t\right) + \sqrt{(N-j+1)j}\frac{\Omega}{2}a_{j-1}\left(t\right) + \sum_{q'} W_{sq'}b_{j,q'}\left(t\right)$$

One integro-differential equation

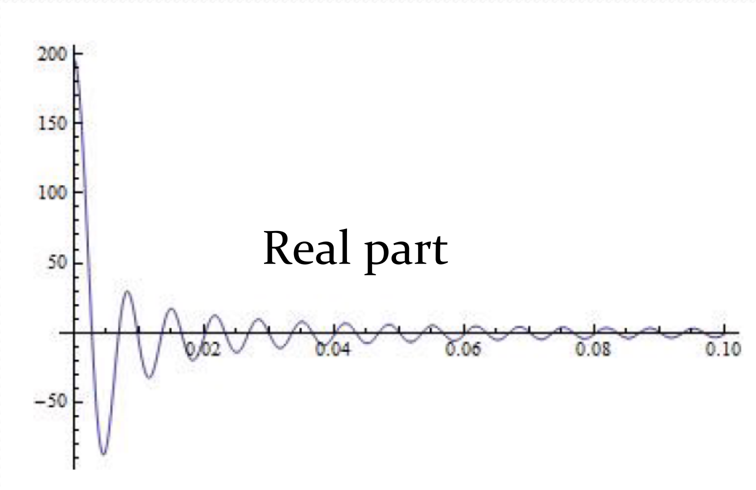
$$i\frac{da_j}{dt} = -\delta j a_j + \sqrt{(N-i)(i+1)}\frac{\Omega}{2}a_{j+1} + \sqrt{(N-j+1)\,j}\frac{\Omega}{2}a_{j-1} + W_{ss}a_j$$
$$-i\sum_q \int_0^t \frac{1}{MN_j}\left[\left[-\frac{d^2}{d\tau^2} - 2i\frac{d}{d\tau}W_{ss} + W_{ss}^2\right]\exp\left(-iW_{qq}\tau\right)\right]\exp\left(i\delta j\tau\right)a_j\left(t-\tau\right)d\tau$$

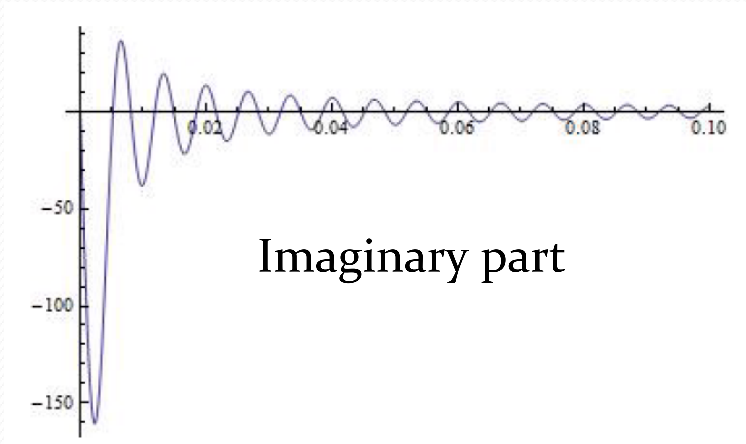
Correlation function

$$-if\left(\tau\right) = -i\sum_q \frac{1}{MN_j}\left[\left[-\frac{d^2}{d\tau^2} - 2i\frac{d}{d\tau}W_{ss} + W_{ss}^2\right]\exp\left(-iW_{qq}\tau\right)\right]$$

Correlation function

Short time, Markovian part





The “mean field term” is exactly compensated!

Long-range term ~ constant

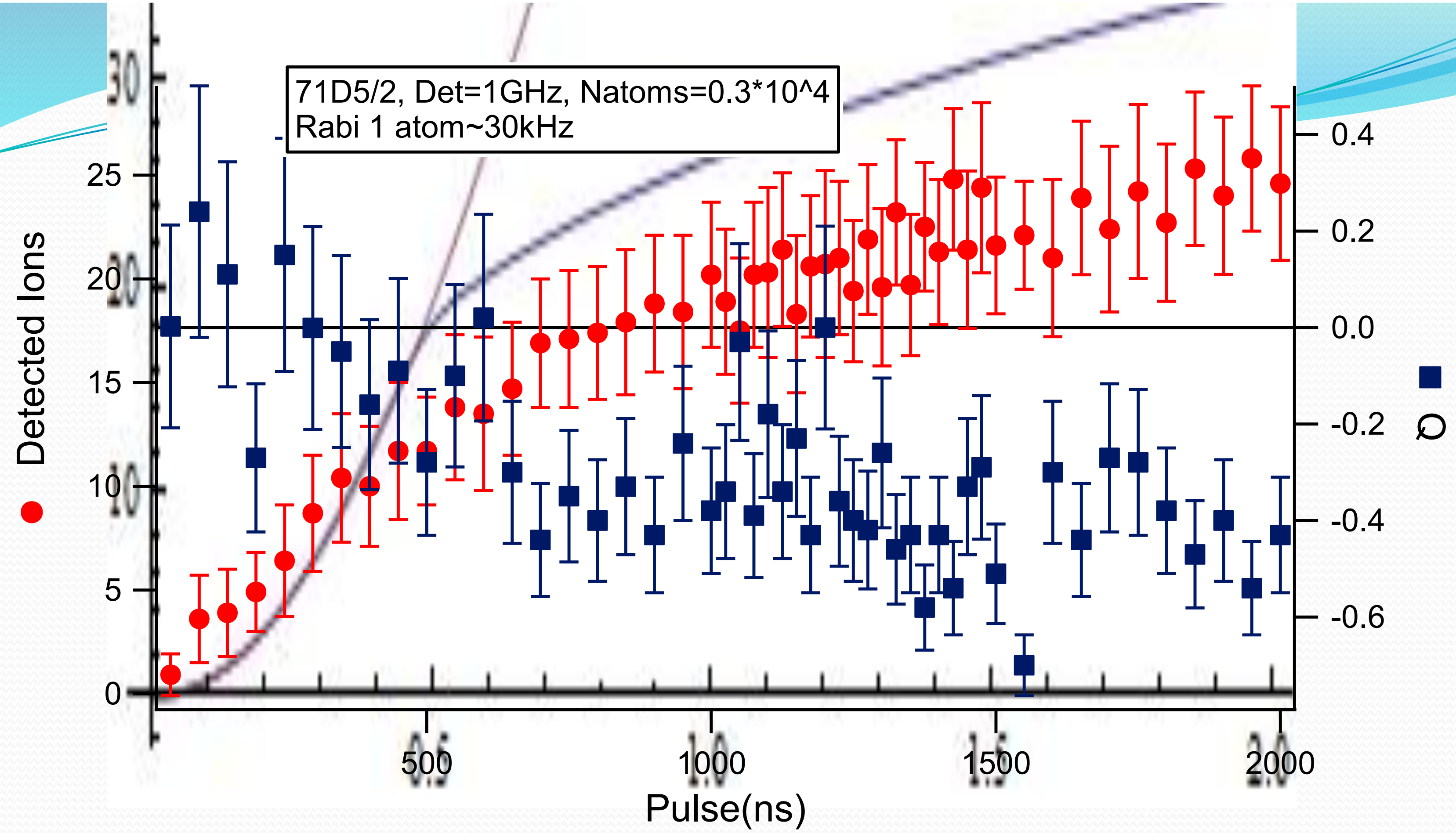
$$i\frac{da_j}{dt} = -\delta j a_j + \sqrt{(N-i)(i+1)}\frac{\Omega}{2}a_{j+1} + \sqrt{(N-j+1)j}\frac{\Omega}{2}a_{j-1} - iW_{ss}^2\int_0^t \exp(i\delta j\tau)\, a_j(t-\tau)\, d\tau$$

We expect symmetrical profiles versus the laser detuning!

The van-der –Waals blockade is described by a coupling of the Dicke symmetrical state with one state (*n*-*s*) which describes the ensemble of the non-symmetrical states

We have neglect up to now the laser excitation of the non-symmetrical states

We introduce it with a population equation by assuming that the equivalent (*n*-*s*) state is essentially a superposition of the less symmetrical state $r=N/2-2j$



25/06/2010

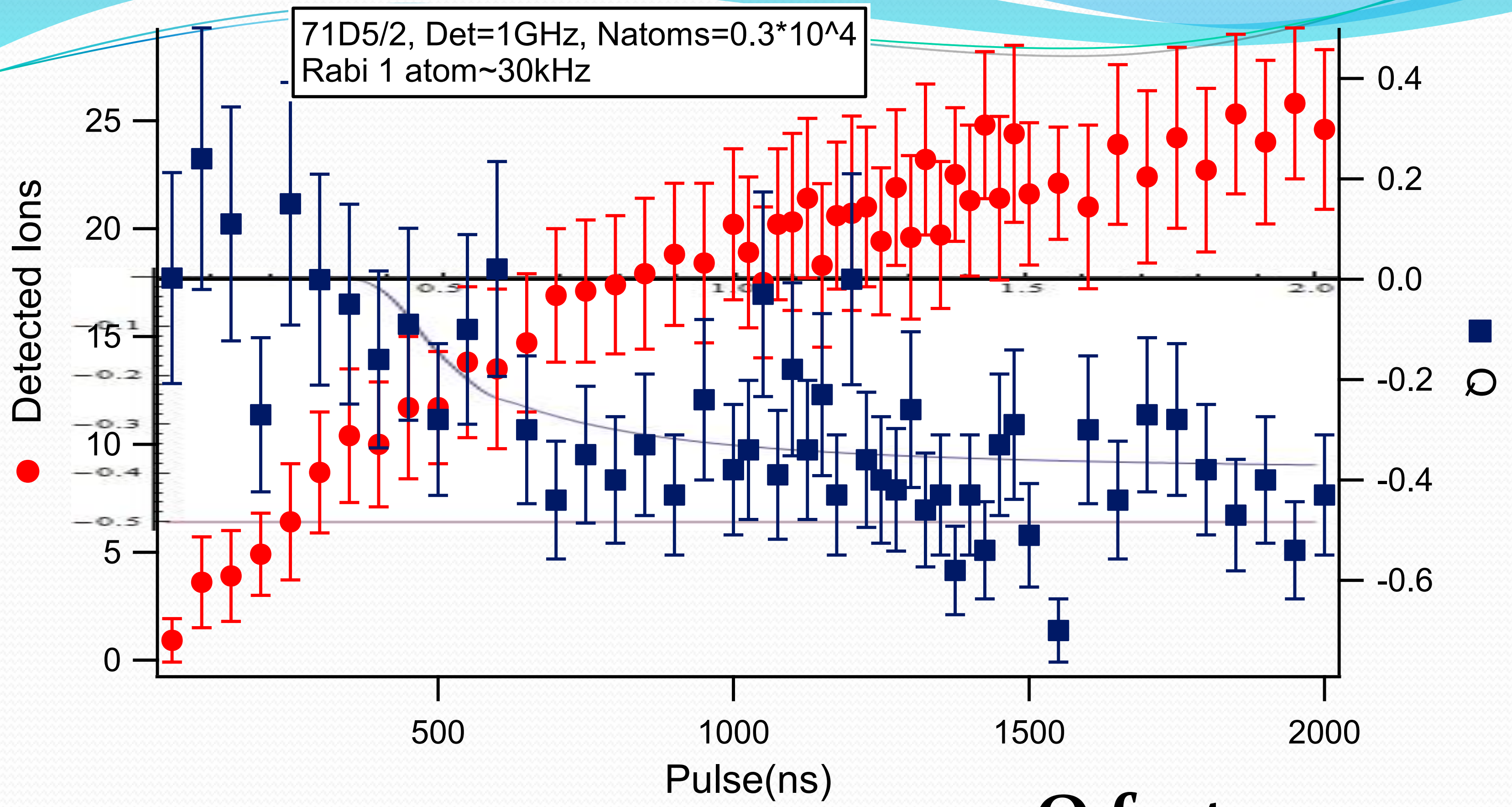
$71d_{5/2}$

Density ≈ 1.2 10^{10} at.cm^{-3}

atoms ≈ 8.10^3

Rabi 1 atom ≈ 45 kHz

Rb $71d_{5/2}$

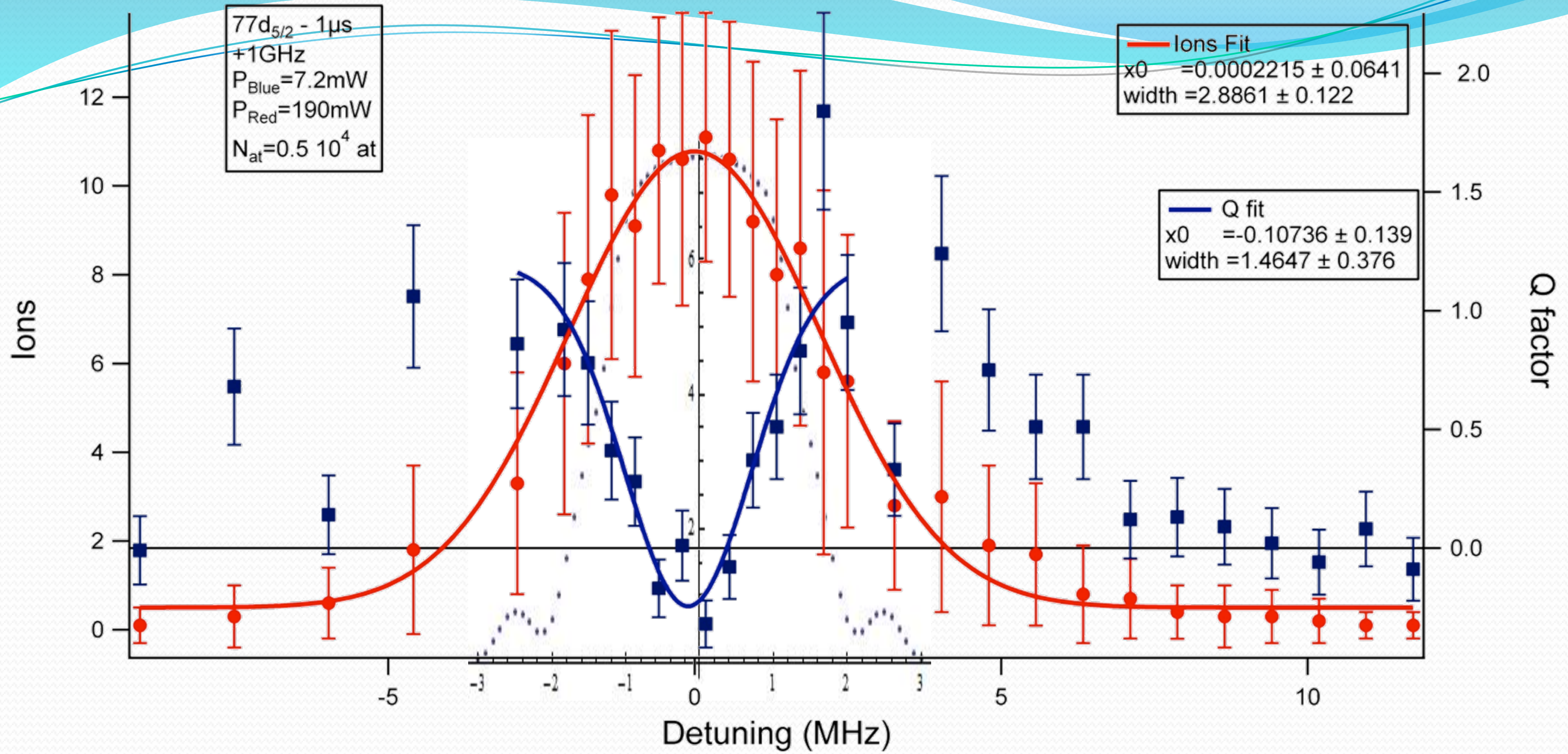


25/06/2010
$71d_{5/2}$
Density ≈ 1.2 10^{10} at.cm^{-3}
atoms ≈ 8.10^3
Rabi 1 atom ≈ 45 kHz

Q factor < 0

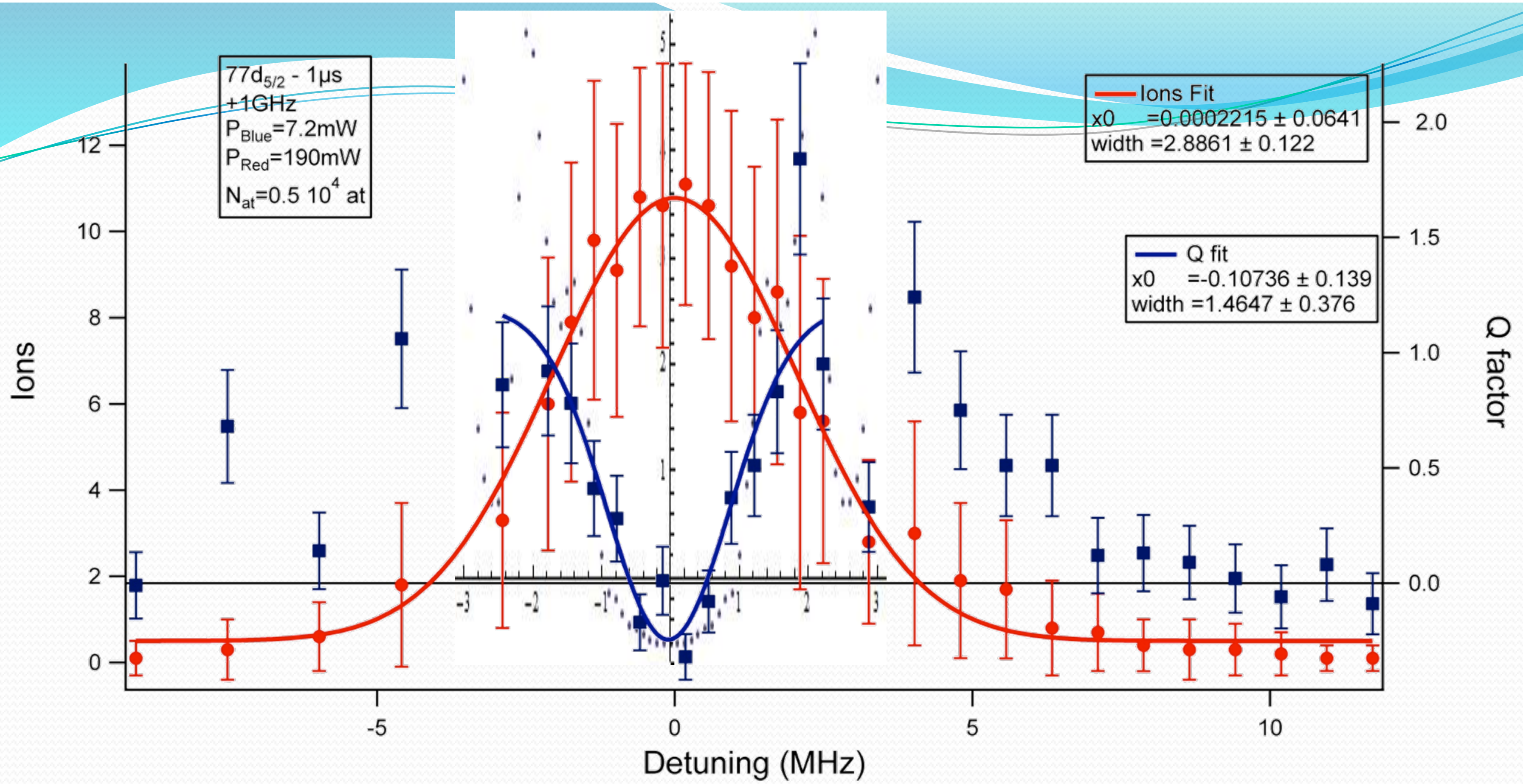
Sub-Poissonian distribution
Correlated events

$$Q = \frac{\langle j^2 \rangle - \langle j \rangle^2}{\langle j \rangle} - 1$$



26/05/2010
$77d_{5/2}$
Density ≈ 7.10^{10} at.cm^{-3}
atoms ≈ 5.10^{3}
Rabi 1 atom ≈ 30 kHz

Rb $77d_{5/2}$

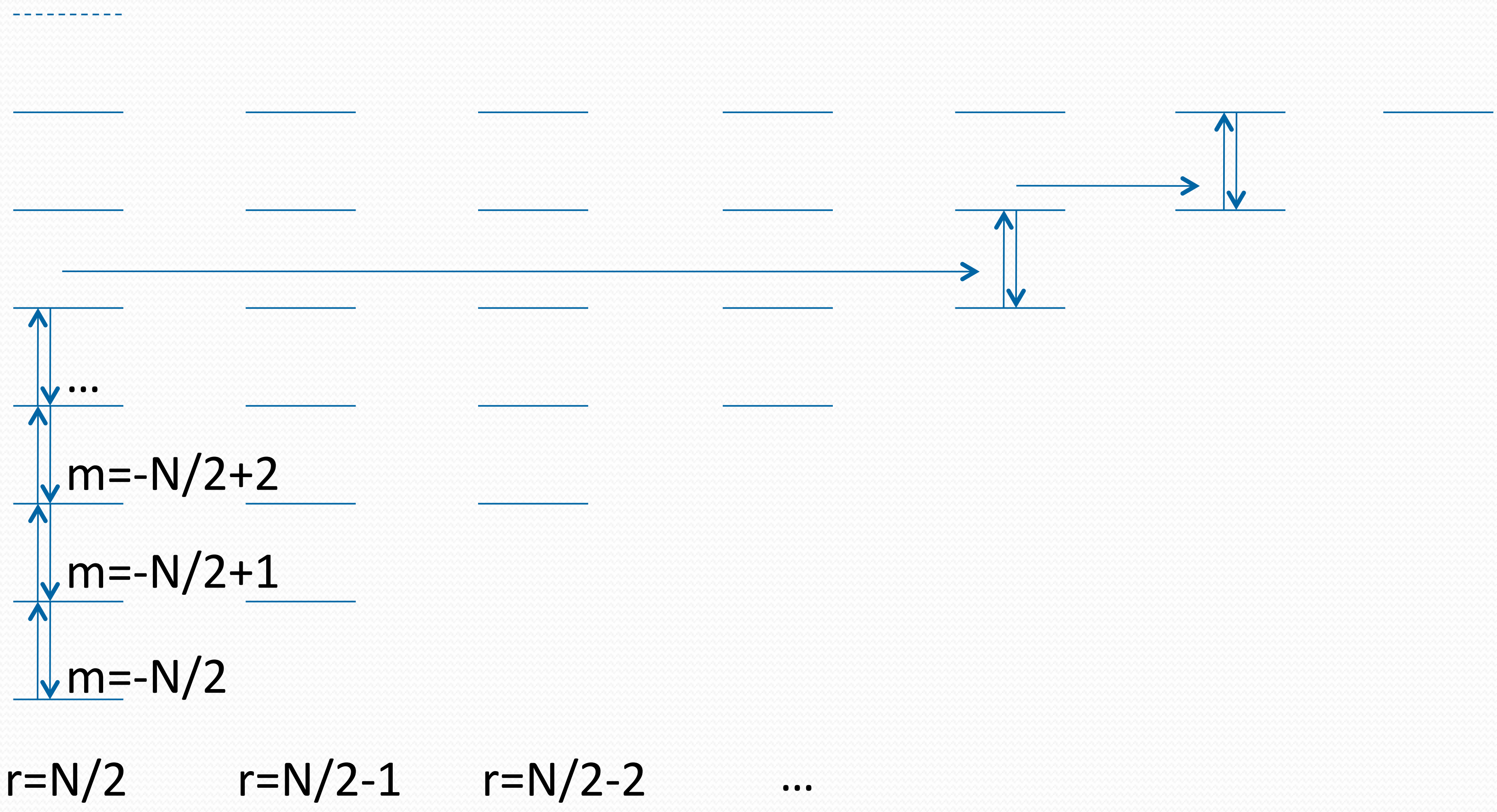


26/05/2010
$77d_{5/2}$
Density ≈ 7.10^{10} at.cm^{-3}
atoms ≈ 5.10^{3}
Rabi 1 atom ≈ 30 kHz

Q factor

$$Q = \frac{\langle j^2 \rangle - \langle j \rangle^2}{\langle j \rangle} - 1$$

Evolution in the Dicke states



Conclusion and a few perspectives

The physics of cold Rydberg atoms is very rich with many open challenges

Two optically active electron atoms

THE END

Thank you for your attention